

Simulation of 2µm single clad thulium-doped silica fiber amplifiers by characterization of the ${}^{3}F_{4} - {}^{3}H_{6}$ transition

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Abstract: We report measurements of absorption, gain, and the lifetime of the transition ${}^{3}\text{H}_{6}$ – ${}^{3}\text{F}_{4}$ for three commercially available thulium-doped single clad silica fibers. These measurements are used in a steady-state simulation of thulium-doped fiber amplifiers (TDFAs). Comparison of simulation and experimental results yield good agreement for a single stage TDFA at 1952 nm and a tandem TDFA at 1910 nm.

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1. Introduction

With the recent progress in transmission system experiments in the 2 μ m band, physical measurements and software tools for the simulation and design of Thulium (Tm)-doped fiber amplifiers (TDFAs) are becoming increasingly important. Accurate measurements of the absorption, gain spectra, and lifetime of the ${}^{3}F_{4} - {}^{3}H_{6}$ transition in silica-doped Tm fibers are required to simulate and compare active fibers from different manufacturers and to rapidly design TDFAs [1]. In this paper, we report the characterization of three commercially available fibers: OFS TmDF200 and iXBlue IXF-TDF-4-125-v1 and v2. Using our physical

measurements and simulation software, we compare our simulations with experimental data for a single stage TDFA at 1952 nm and a two stage TDFA at 1910 nm. Good agreement is found between the simulations and experimental results. Our model can be used to design TDFAs for telecommunications or LIDAR applications.

2. Fiber characteristics and measurements

Characterizing the fibers requires three fundamental measurements: transition lifetimes, absorption coefficients, and gain coefficients.

2.1 Lifetime measurements

The first measurement conducted was the fluorescence lifetime (τ_{obs}) of the ${}^{3}F_{4}$ level. The lifetimes were determined using an in-band modulated pump source. As shown in Fig. 1, a 1.55 µm pump pulse from a Fabry-Perot was launched using a 3 dB coupler into the Tm-doped fiber (TDF), which is less than 10 cm in order to prevent gain. The pump was modulated at 100 Hz, with a rectangular modulation width of 500 µs and 15 mW peak power. Spontaneous light from the fiber was emitted around 1.8 µm and the backward emission was monitored using an amplified photodiode. The observed curve is a sum of two exponentials as described in the literature [2–4]: a long decay ($\tau_{1}\approx$ 450-750 µs) and a short decay ($\tau_{2}\approx$ 100-300 µs). This decay behavior is attributed to either inhomogeneity of the environment or ion-ion interactions [5–7]. The long decay τ_{1} is usually considered as the fluorescence lifetime of the level, used as a parameter in the simulation. The amount of power launched was varied by more than 10 dB to make sure the lifetimes measured were independent of the pump power. The measurement of lifetime for both OFS and iXBlue v2 are shown in Fig. 2. A fit to the data leads to $\tau_{1} = 650 \pm 20$ µs for the OFS fiber, 750 \pm 50 µs for the iXBlue v1 and 475 \pm 10 µs for v2.



Fig. 1. Experimental setup for measurement of transition lifetimes.



Fig. 2. Measured ${}^{3}F_{4}$ lifetime curve with its two exponential fit for the OFS and iXBlue v2 fibers.

2.2 Absorption coefficient measurements

The absorption coefficient was measured with a tunable laser and an ASE source using a cutback method. Low power sources were used to measure the true or unsaturated absorption coefficient of the transition. Multiple cut-backs were performed with sample lengths of a few centimeters to multiple meters to measure the full spectrum. The data were fitted using a sum of 3 to 4 Gaussians as shown in the solid blue lines of Fig. 4, including the measurement error calculated at 1640 nm. The fit allowed to smoothly fit the data at long wavelengths (λ >2000 nm).

2.3 Gain coefficient measurements

The small signal gain coefficient of the fibers was determined using three different methods. The first method relies on the saturated fluorescence technique [8]. The setup used is displayed in Fig. 3. An OSA is connected on port P1 while port P2 is left unconnected. The fiber under test (a few centimeters long) was in-band pumped with couple watts of CW light at $\lambda_p = 1567$ nm through a broadband wavelength division multiplexer (WDM). This pumping wavelength is assumed to allow us to optimally invert the population ($\alpha(\lambda_n) >> g^*(\lambda_n)$ where $\alpha(\lambda)$ is the absorption coefficient of the transition and $g^*(\lambda)$ is the gain coefficient of the transition) which then gives us a good approximation of the transition gain. The pump power (P_{p}) was increased until the backward fluorescence, monitored with the OSA, no longer changed as a function of pump power level: $P_p >> P_{p \text{ sat}}$ where $P_{p \text{ sat}}$ is the saturation power at the pumping wavelength. The spontaneous output power $P_{sp}(\lambda,L)$ is given by Eq. (1) [9] where $T(\lambda)$ is the transmission function of the setup to the OSA, h is the Plank constant, c_0 is the speed of light in vacuum, $\Delta \lambda$ is the resolution of the OSA, and L is the sample length. The experimental data were corrected using the loss of the setup and fitted with a sum of 3 to 4 Gaussian curves. Fitting the data allows us to remove the water absorption from the measured spectrum and also to recover the gain at wavelengths below 1650 nm (WDM1 had only an cross talk between the pump and the fluorescence of ~ 20 dB). The fits are shown in the plots of Fig. 4 as dotted orange lines, including the calculation of the measurement error at 1820 nm.



Fig. 3. Experimental setup for measurement of the gain coefficient transition.

$$P_{sp}(\lambda,L) = T(\lambda) \cdot \frac{2h.c_0 \cdot \Delta \lambda}{\lambda^3} \Big[\exp(g^*(\lambda) \cdot L) - 1 \Big].$$
(1)

The second method relies on launching small signals around 2 μ m through port P1 in the setup displayed in Fig. 3 and directly measuring their amplification [10]. The amplification is monitored with an OSA connected on port P2. When the pump is off, the signal only sees the absorption through the sample. When the pump saturates the sample, the signal sees only the gain. The difference between the two states gives us the small signal gain since we already know the small signal absorption at the signal wavelength. This equation is given by Eq. (2) where P_{ON} is the output power when the pump is on and P_{OFF} is the output power when the pump is off. This method is expected to give the most accurate results, but its flexibility is limited by the available wavelengths of the single frequency signal lasers employed in the measurement. These measurements are plotted as purple points in Fig. 4.

$$g^{*}(\lambda) = \alpha(\lambda) + \frac{P_{ON}(\lambda) - P_{OFF}(\lambda)}{L}.$$
 (2)



Fig. 4. Absorption, gain from saturated fluorescence, McCumber gain, and directly measured small signal amplification for OFS and iXBlue v1 and v2 commercial Tm-doped fibers.

The third method uses McCumber theory on the fitted absorption coefficient [11]. Here the calculation is done assuming room temperature at 25 °C. This relation is given in Eq. (3) where k_B is the Boltzmann constant, T is the ambient temperature in Kelvin, and λ' is the wavelength at which the gain and the absorption cross $(\alpha(\lambda') = g^*(\lambda'))$. This wavelength is determined using the intersection of the fitted absorption and the previously fitted gain, see Table 1. These calculations are shown in Fig. 4 as a dashed green line.

$$g^{*}(\lambda) = \alpha(\lambda) . \exp\left[-\frac{h.c_{0}}{k_{B}.T}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)\right].$$
(3)

We note that in the plots of Fig. 4, the direct measurements of the gain coefficients at fixed wavelengths agree quite well with the gain coefficient curves obtained with the saturated fluorescence method. Agreement with gain coefficient curves calculated using McCumber theory is not as good. For this reason we employ the saturated fluorescence gain curves in our numerical simulations. However we also note that McCumber theory yields a good approximation of the gain coefficient. This theory can be useful in cases where it is difficult to directly measure the gain coefficient, for example, in highly doped single or double clad fibers.

Using the data from Fig. 4, spectral differences between the iXBlue and OFS fibers are summarized in Table 1 where λ_{peak} is the peak wavelength and FWHM is the full width half maximum. The iXBlue fibers have higher doping levels (1550 nm absorption ≈ 2.4 and 3.2

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times greater) than the OFS fiber. Differences in the core composition, co-dopants, and codoping ratios yield differences in the optical spectral characteristics.

Fiber		iXBlue v1	iXBlue v2	OFS
α(λ)	$\lambda_{peak} (nm)$	1623	1643	1645
	FWHM (nm)	183	195	180
g*(λ)	$\lambda_{peak} (nm)$	1791	1801	1818
	FWHM (nm)	312	301	298
λ' (nm)		1736	1748	1739

Table 1. Summary of spectral data for iXBlue and OFS fibers.

2.4 Summary of fiber parameters

Our simulation tool [11] is based on a simplified three level model taking into account the levels: ${}^{3}H_{6}$, ${}^{3}F_{4}$ and ${}^{3}H_{4}$, allowing to simulate single or double clad fibers with different pumping schemes. In Table 2 we present a comprehensive list of the parameters used in the simulation model. We note first that the OFS and iXBlue fiber v2 have significantly different core diameters and numerical apertures. The overlap factor is defined as the overlap between the dopant distribution and the core propagating mode. The overlap factors were calculated using a step index approximation. Values of the ${}^{3}F_{4}$ lifetimes are from our measurements, while ${}^{3}H_{4}$

Table 2. List of parameters used in the simulation model.

Parameter	Fiber values			Reference	
Manufacturer		OFS	iXBlue		/
Ref		TmDF200	4µm v1	4µm v2	/
Core diameter (µm)		4	4	5.3	Manufacturer datasheet
Core NA (u.a.)		0.26	0.27	0.17	
³ F ₄ lifetime (μs)		650	750	485	
$\alpha(\lambda) (dB.m^{-1})$	@1550 nm @1952 nm	23.4 1.68	55.7 1.55	76 3.98	Measured
$g^{*}(\lambda) (dB.m^{-1})$	@1550 nm @1952 nm	8.5 32	13.6 43.8	16.7 80.1	
Overlap factor (u.a.)	@1550 nm @1952 nm	0.74 0.58	0.76 0.61	0.65 0.43	Calculated
Doping level (m ⁻³)		8.4×10^{25}	1.23×10^{26}	2.56x10 ²⁶	[1]/Calculated
³ H ₄ lifetime (μs)		12	14.2		[1]/[14]
$k_{3011} (m^3.s^{-1})$		3x10 ⁻²³	1.8×10^{-22}		[12]/[14]
$k_{1130} (m^3.s^{-1})$		2.4x10 ⁻²⁴	1.51×10^{-23}		
Background loss (dB.km ⁻¹)		100		[14]	

lifetimes are taken from published results [1,13]. The measured values of absorption and gain at 1550 nm and 1952 nm confirm that pumping the fibers in the 1550 nm band is an efficient way to produce gain in the 1950 nm band. Background loss is set to 100 dB/km based on data in the literature [14]. The model also takes in account two ion-ion interaction processes: ${}^{3}H_{6}$, ${}^{3}H_{4} = > {}^{3}F_{4}$, ${}^{3}F_{4}$ (resp. ${}^{3}F_{4}$, ${}^{3}F_{4} = > {}^{3}H_{6}$, ${}^{3}H_{4}$) which is a cross relaxation effect (resp. energy transfer up-conversion effect) represented by the coefficient k_{3011} (resp. k_{1130}). These coefficients values were taken from the literature [12,14]. These parameters aren't important for single clad in-band pumped amplifiers but necessary to simulate double clad amplifiers. For the OFS fiber we employ a doping level reported by Agger and Povlsen [1]. For the iXBlue fibers we calculated the doping level by comparison with the OFS fiber data,



assuming that the cross sections for the OFS fiber are accurate for Tm-doped silica fibers with relatively low doping concentrations.

3. Single stage TDFA experiments and simulation

We now turn to comparisons of experiment and theory for a single stage TDFA. Using the parameters we measured on the OFS fiber and data from Table 2, we simulated and evaluated a core-pumped amplifier operating at 1952 nm. This single stage amplifier is shown in Fig. 5 where F1 is a 7 m length of OFS TmDF200 co- and counter-pumped at 1550 nm by two DFB diodes (P1 & P2), each delivering around 200 mW into the active fiber. The input and output are both isolated to prevent spurious lasing. The input and output loss of the signal and pump were estimated from the datasheet of the components. Data displayed are relative to the input and output of the active fiber.



Fig. 5. Experimental setup for study of a single stage TDFA.

Three simulation cases are considered: with or without the ion-ion interactions for the doping level from the literature, and then with ion-ion interactions for a doping level adjusted to get better agreement with the experimental data. We define the gain of the amplifier (G) through Eq. (4) where P_{in} is the input signal power and P_{out} is the output signal power. We also define the noise figure (NF) through an optical method [9] see Eq. (5) where $P_{ASE}^{forward}(\lambda)$ is the ASE propagating forward below the signal.

$$G(\lambda) = \frac{P_{out}(\lambda)}{P_{in}(\lambda)}.$$
(4)

$$NF(\lambda) = \frac{1}{G(\lambda)} \left[1 + \frac{P_{ASE}^{forward}(\lambda) \cdot \lambda^3}{h \cdot c_0^2 \cdot \Delta \lambda} \right].$$
(5)



Fig. 6. Output power as a function of the counter-pumping power for a 7 m OFS amplifier at 1952 nm.

Our first comparison of experiment and simulation is shown in Fig. 6. Here we plot the 1952 nm output power of the single stage amplifier as a function of the 1550 nm counterpump power launched into the 7 m OFS Tm-doped fiber. The co-pump power is constant at 220 mW, and the signal input power is 2.1 dBm. The measured output powers are shown as points on the graph. A straight line fit to the data (dotted blue line) yields an experimental slope efficiency ($\eta = \Delta P_{out}/\Delta P_p$) of 54%. Three simulated curves are also plotted in Fig. 6. The solid orange curve is calculated for a doping level of 8.4×10^{25} m⁻³ and no ion-ion interactions, while the dashed green curve includes ion-ion interactions for the same doping level. For these two curves, the difference between theory and experiment is a maximum of 0.8 dB. We then adjusted the doping level to 9.1×10^{25} m⁻³ and repeated the simulations with ion-ion interactions. The result is the dash-dotted yellow curve which agrees with the output power data to within 0.5 dB. Up to 140 mW of signal output power at 1952 nm could be extracted from this amplifier at full pump power. We note that the difference between the simulation and the measured data is a function of the pump power; this behavior is under study.



Fig. 7. Signal gain as a function of the input signal power at 1952 nm and full co- and counterpropagating pump powers.

The next experiment was to run the amplifier at full pump power and vary the input signal power using an attenuator in between the input laser and the amplifier. The amplifier gain was measured as a function of signal input power and the results are plotted in Fig. 7. Greater than 40 dB of signal gain was demonstrated for an input signal power of -35 dBm as illustrated by the experimental data points. The solid orange line is a simulation without ion-ion interactions using a doping level of 8.4×10^{25} m⁻³, while the dashed green line is for the same doping level including ion-ion interactions. The dash-dotted yellow curve is a simulation including ion-ion interactions for a doping level of 9.1×10^{25} m⁻³. The difference between simulation and experiment is found to be again less than 0.7 dB for the best fit and is independent of the input signal power.

The noise figure was also simulated and measured as shown in Fig. 8, and found to be experimentally 3.1 to 3.5 dB for input signal powers below -5 dBm. For the simulation, we simulated two cases: first with a monochromatic input signal spectrum and second using the measured spectrum of the input seed. The three simulations for noise figure follow the same descriptions as in Fig. 6. It is evident that simulation and experiment for the noise figure agree well for input signal powers below -5 dBm for either the monochromatic and measured input spectrum. The differences between experiment and simulation for input powers of 0 dBm and above are under investigation.



Fig. 8. Experimental and simulated noise figures for the single stage TDFA.

4. Two stage TDFA experiments and simulation

A two stage amplifier configuration was also studied to provide a further comparison of experiment and simulation. As shown in Fig. 9, Stage 1 of the tandem amplifier is the single stage TDFA used in Fig. 5. Stage 2 is a power amplifier which boosts the output signal from Stage 1 using another single clad Tm-doped fiber F2: either 5.3 m of IXF-TDF-4-125-v1 or 5 m of TmDF200. A fiber laser (P3) at 1567 nm counter-pumps F2 and delivers up to 3.2 W of internal pump power. The first stage pumps are operated at full power. Input and output signals are measured at the input of F1 and the output of F2 respectively. The signal wavelength for the tandem amplifier studies is 1910 nm.

We note that including the ion-ion interactions and varying the doping level had relatively little effect on the simulations for the single stage amplifier in Section 3. For this reason, we chose to simulate the dual stage amplifier without ion-ion interactions and with a doping level of $8.4 \times 10^{25} \text{ m}^{-3}$.



Fig. 9. Setup for the two stage TDFA.



Pump power @ 1567 nm (W)

Fig. 10. Output Power vs. Pump Power for the two stage TDFA.



Fig. 11. Gain and Noise Figure vs. Signal Input Power for the two stage TDFA.

Figure 10 is a comparison of simulation and experiment for output power vs. pump power for the two stage amplifier, with an input signal power of 1.34 dBm at 1910 nm. By examining the differences between experiment (points) and simulations (solid lines) we find that the simulated output power for the iXBlue fiber is 1 dB greater than the experimental measurements. Simulation for the OFS fiber is 0.5 dB greater than experiment. This represents good agreement between the theory and the data. The differences between simulation and experiment are under study, especially with respect to the different results for the two fiber types.

In Fig. 11 we compare simulation and experiment for the two stage amplifier with respect to the parameters of gain and noise figure. Once again the agreement between theory and data is relatively good, although the calculated gain is somewhat greater than the experimental gain for input signal powers less than -20 dBm. Measured noise figure values also diverge from the simulated values for input signal powers less than -15 dBm. Nevertheless we find that for input signal powers > -15 dBm, our simulation agrees relatively well with the experimental measurements. The origin of the differences for $P_{in} < -15$ dBm is currently under study.

5. Discussion

Our measurements demonstrate our ability to precisely characterize a TDF, and also confirm Agger's and Povlsen's data [1] on the OFS fiber for the absorption coefficient and the lifetime decay. We observe a peak emission to peak absorption ratio of less than 1, according to both the saturated fluorescence method and the McCumber method. Overall, reasonable agreement is found between the two methods, but some differences at higher and lower wavelengths need to be explained. We also observe a good agreement between the saturated fluorescence method and the small signal amplification at 2050 nm, confirming the gain measured using the saturated fluorescence. The measurement of the saturated fluorescence is usually performed at a higher energy transition [8,9] instead of using a resonant pumping ($\lambda_p = 1567$ nm). Nevertheless the pumping wavelength chosen allows to provide a good approximation as demonstrated by our measurement.

Using our physical measurements of the OFS fiber and a value of $N = 8.4 \times 10^{25} \text{ m}^{-3}$, our simulation results of the single stage 7 m OFS amplifier were within 0.8 dB of experiment, when ion-ion interaction is taken into account. These results are encouraging but suffer from inaccuracies in parameters such as the doping level and the background loss. As an illustration, by adjusting the doping parameter from $8.4 \times 10^{25} \text{ m}^{-3}$ to $9.1 \times 10^{25} \text{ m}^{-3}$ we obtained simulation results that were within 0.5 dB of experimental results. Our simulations of output power, gain, and noise figure for the two stage amplifier also agree relatively well with the experimental data, with measured output power within 1 dB and 0.5 dB for the iXBlue and OFS fibers, respectively.

We observe that a better knowledge of the fiber parameters is needed to simulate the fiber in a more accurate and precise way. Therefore we intend to expand the scope of our experimental measurements to more accurately determine the doping level N, the ion-ion interaction coefficients k_{3011} and k_{1130} , and the background losses of the OFS and iXBlue fibers. We also intend to extend our signal wavelength measurements to the entire Thulium transmission band in order to more fully evaluate our model and our simulation software.

6. Summary

In this paper we established a characterization method for measuring three important parameters in commercially available OFS and iXBlue Tm-doped fibers. These parameters are the absorption, the gain, and the lifetime of the ${}^{3}F_{4} - {}^{3}H_{6}$ transition. Our measurements produced data that were used in a simulation tool to model single clad Tm-doped fiber amplifiers.

We first validated the use of our parameters by simulating a single stage TDFA amplifier with 7 meters of the OFS fiber co- and counter-pumped at 1550 nm. The simulation at 1952 nm signal wavelength showed agreement to within 0.5 dB of the experimental output power data. In addition, both the simulation of small signal gain and the experimental data showed

up to 40 dB of internal small signal gain. Simulation and measurement of the small signal internal noise figure yielded values close to the quantum limit of 3 dB.

We then validated our parameters for a two stage TDFA using combinations of OFS/OFS and OFS/iXBlue fibers. The simulations at 1910 nm showed relatively good agreement with the experimental data, with differences of 1.0 and 0.5 dB in output power for OFS/iXBlue and OFS/OFS configurations, respectively. Measurements of small signal gain and noise figure also agree well with simulations for $P_s > -15$ dBm. The origin of differences between experiment and theory for $P_s < -15$ dBm is under study.

We note that the agreement between simulation and experiment is sufficiently good for our software to be used as a simulator for the accurate design of multi-stage TDFAs.

In future work, we plan to make more accurate measurements of number density, k_{3011} and k_{1130} , and background loss for the OFS and iXBlue fibers. We will also measure the performance of the TDFAs over the entire signal amplification band to more fully compare our theory and simulations to the experimental data.

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